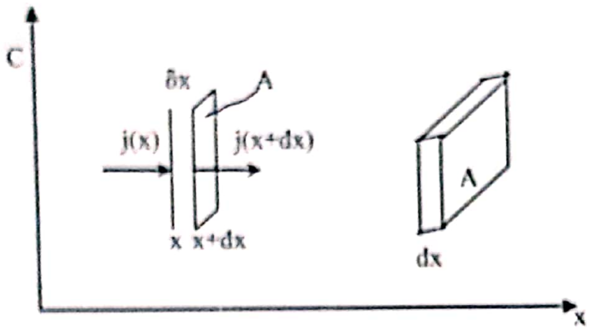
4: Diffusion: *Fick's second law*

Today's topics

- Learn how to deduce the Fick's second law, and understand the basic meaning, in comparison to the first law.
- Learn how to apply the second law in several practical cases, including homogenization, interdiffusion in carburization of steel, where diffusion plays dominant role.

Continued from last lecture, we will learn how to deduce the Fick's second law, and understand the meanings when applied to some practical cases.

Let's consider a case like this



We can define the local concentration and diffusion flux (through a unit area) at position "x" as:

c (x,t), J(x)

we have

$$\delta c(x) = \frac{[j(x) - j(x+dx)]\delta t A}{A dx}, \qquad J(x+dx) = J(x) + \frac{\partial j}{\partial x}\delta x$$

then, we have

$$\frac{\partial c(x,t)}{\partial t} = -\frac{\partial j}{\partial x}$$

form the first law: $J = -D \cdot \frac{dc(x)}{dx}$

then, we have

$$\frac{\partial c(x,t)}{\partial t} = -\frac{\partial j}{\partial x} = D \cdot \frac{\partial^2 c}{\partial x^2} \qquad (1)$$

his is the Fick's second law.

In three-dimensional space, it can be written as:

$$\frac{\partial c}{\partial t} = D \cdot \nabla^2 c .$$

At steady (equilibrium) state, we have

$\partial c(x,t)/dt = 0$ (meaning no concentration change)

Then, solving Eq. (1) gives

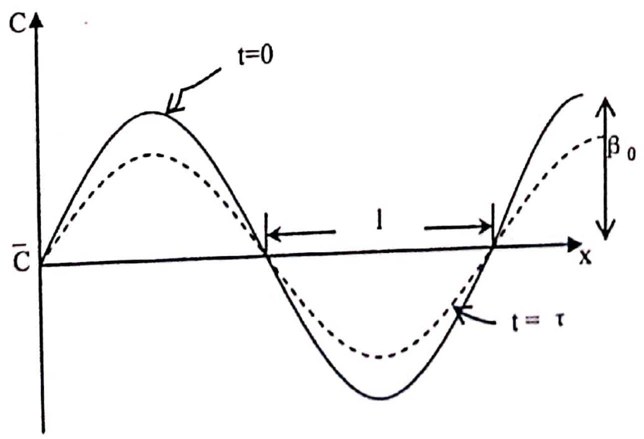
$-D \cdot \frac{dc}{dx} = J =$ constant --- back to the Fick's first law.

So, Fick's first law can be considered as a specific (simplified) format of the second law when applied to a steady state.

Now, let's consider two real practical cases, and see how to solve the Fick's second law in these specific cases.

Case 1. Homogenization: (non-uniform → uniform)

Consider a composition profile as superimposed sinusoidal variation as shown below, where the solid line represents the initial concentration profile (at t=0), and the dashed line represents the profile after time τ.



At t=0, $c = \bar{c} + \beta_0 \sin\frac{\pi x}{l}$

with the Fick's second law, $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$, where D is teh diffusion coefficient, a constant.